Your Signature ____________________

Student I.D.# | | | | | | | | |

This is a closed book exam. Calculators are permitted. There are seven problems. Maximum possible score is 100 points. **Show all your work.** *Partial credit will be given for partial solutions. Correct answers with insufficient or incorrect work will not get any credit.*

Score

1. (20)	
2. (20)	
3. (20)	
4. (20)	
5. (20)	
6. (20)	
Total. (120)	

Extra sheets attached(if any):____________________

Please do not open this booklet until you are asked to do so.

1. Suppose there are three bowls and three chips. For each of the colours red, blue,and green there is one chip and one bowl painted that colour. The three chips are placed in the three bowls randomly, so that each bowl contains exactly one chip. Assume that all possible placements are equally likely. Please answer the following questions:

(a) Describe the sample space Ω of possible placements of the chips. How many elements are there in the sample space Ω ?

(b) Let X be the number of bowls containing a chip of the same colour as that bowl. Find the probability mass function for X.

(c) Suppose this experiment is repeated eight times. That is, each time the chips are removed and then placed back randomly, one per bowl. What is the probability that in exactly two of the 8 trials, no chip was placed in a bowl of the same colour ?

2. Suppose that 15% of the engines manufactured on a certain assembly line are defective. The engines are randomly selected one at a time and tested.

(a) Let X denote the number of the trial at which the first defective engine is found. What is the probability mass function of X ? What is E(X) ?

(b) Let Y denote the number of the trial at which the tenth defective engine is found. What is the probability mass function of Y ?

(c) Using information from part 2(a) and part 2(b), determine $E(Y)$.

3. The number of people N that arrive at a polling station has a Poisson distribution with mean λ. Each person independently votes for Tree party with probability p. Let X be the total number of people that voted for Tree party. Find the distribution of X.

4. Waiting times at a service counter in a pharmacy are exponentially distributed with a mean of 5 minutes.

(a) What is the probability that one customer has to wait for more than 5 minutes ?

(b) Let 50 customers come to the service counter in a day. Let S_{50} be the number of customers that wait for more than 10 minutes.

  (i) Write out the probability mass function of S_{50}.

  (ii) Using the central limit theorem, approximate the probability that at least half of the customers that arrive in a day must wait for more than 10 minutes. Please explain clearly how the central limit theorem is used in the solution.